Outline

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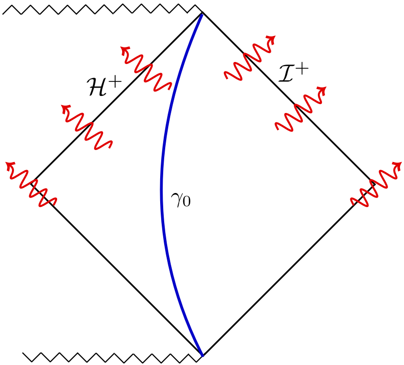
Typical calculation at first order [Capra community]



- approximate the source orbit as a bound geodesic
- impose outgoing-wave BCs at $\mathcal{I}^+$ and $\mathcal{H}^+$
- unphysical on long timescales. Breaks down on dephasing time $t \sim M/\sqrt{\epsilon}$

Solving the first-order field equations [Capra community]

- directly solve Lorenz-gauge linearized Einstein equation for $h^{(1)}_{\mu\nu}$
  - advantage: most expressions for singular field are in Lorenz gauge. Easily separated into ℓm modes in Schwarzschild
  - disadvantage: not separable in Kerr
- solve Regge-Wheeler and Zerilli equations for master functions $\Psi_{\ell m}$

  $\Rightarrow h^{(1)\ell m}_{\mu\nu} \sim \partial\Psi_{\ell m}$
  - advantage: simple scalar ODEs
  - disadvantages: not defined in Kerr. Pathological singularities away from particle
- solve Teukolsky equation for curvature scalar $\psi_4^{\ell m\omega}$

  $\Rightarrow h^{(1)\ell m\omega}_{\mu\nu} \sim \partial\partial\iiiint \psi_4^{\ell m\omega}$
  - advantage: fully separable into $\ell m\omega$ modes in Kerr
  - disadvantage: pathological singularities away from particle

Multiscale expansion [Miller & AP; AP & Wardell; Flanagan, Hinderer, Moxon, AP]

- all time dependence in $h_{\mu\nu}$ follows from puncture's motion and black hole's evolution
- recall $(z^\mu, u^\mu) \to (\tilde{\varphi}_A, \tilde{J}^A)$. Define full set of system parameters $\mathcal{J}^A \sim (\tilde{J}^A, M, a)$

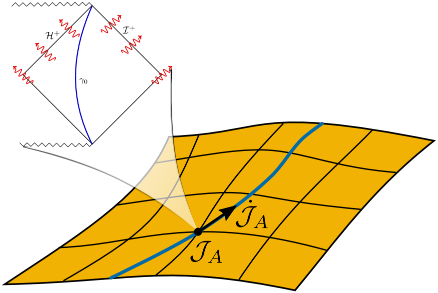
$$\frac{d\tilde{\varphi}_A}{dt} = \Omega_A(\mathcal{J}^B)$$
$$\frac{d\mathcal{J}^A}{dt} = \epsilon \tilde{G}^A_{(1)}(\mathcal{J}^B) + \epsilon^2 \tilde{G}^A_{(2)}(\mathcal{J}^B) + O(\epsilon^3)$$

- treat $h_{\mu\nu}$ as function on extended manifold: in spacetime coords (t, x^i),

$$h_{\mu\nu} = \epsilon h^{(1)}_{\mu\nu}(\tilde{\varphi}_A, \mathcal{J}^A, x^i) + \epsilon^2 h^{(2)}_{\mu\nu}(\tilde{\varphi}_A, \mathcal{J}^A, x^i) + O(\epsilon^3)$$

- in Einstein equations, $\dfrac{\partial}{\partial t} = \Omega_A \dfrac{\partial}{\partial \tilde{\varphi}_A} + \dfrac{d\mathcal{J}^A}{dt} \dfrac{\partial}{\partial \mathcal{J}_A}$

Multiscale expansion [Miller & AP; AP & Wardell; Flanagan, Hinderer, Moxon, AP]



- Fourier series:

$$h^n_{\mu\nu} = \sum_{k^A} h^{n,\Omega_k}_{\mu\nu}(\mathcal{J}^A, x^i)e^{-ik^A\varphi_A}$$

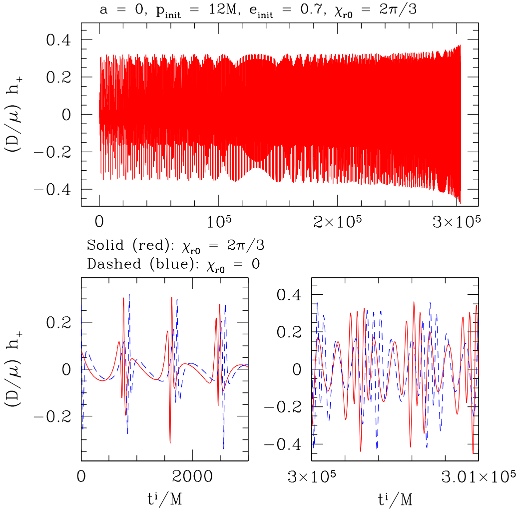
$$\Omega_k := k^A\Omega_A$$

- solve field equations for amplitudes $h^{n,\Omega_k}_{\mu\nu}$ on grid of $\mathcal{J}^A$ values

- millisecond waveform generation when combined with FastEMRIWaveforms tools [Katz, Chua, Speri, Warburton, Hughes]

Outline

Adiabatic waveforms

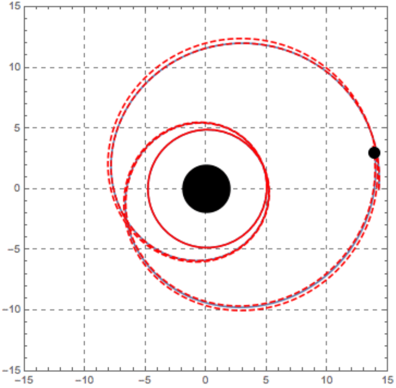


[Hughes et al]

Outline

1. Lecture 1: gravitational wave astronomy, the two-body problem, and self-force theory
2. Lecture 2: the local problem: how to deal with small bodies
3. Lecture 3: the global problem: orbital dynamics in Kerr
4. Lecture 4: the global problem: black hole perturbation theory
   - Multiscale expansion of the field equations
   - Results at 0PA order
   - Results at 1PA order

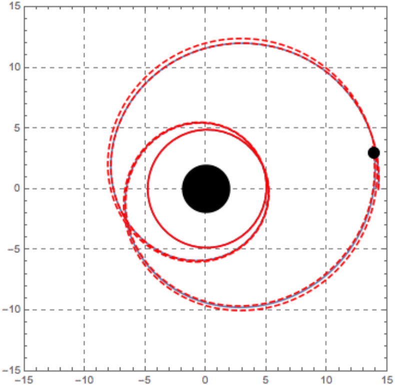
Complete first-order self-force



[image courtesy of Warburton]

- complete inspirals simulated in Schwarzschild using full F_1^μ (including spin force) [Warburton et al]
- and F_1^μ has been computed on generic orbits in Kerr [van de Meent]
- but still need F_2^μ for post-adiabatic inspiral

Complete first-order self-force



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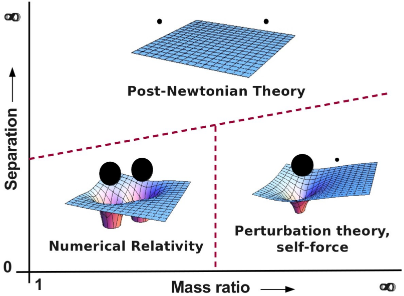
First-order results: improving other binary models

- PN and EOB models have been improved using data for effects of the self-force
- fixed high-order PN coefficients, completed 4PN, settled controversy at 4PN

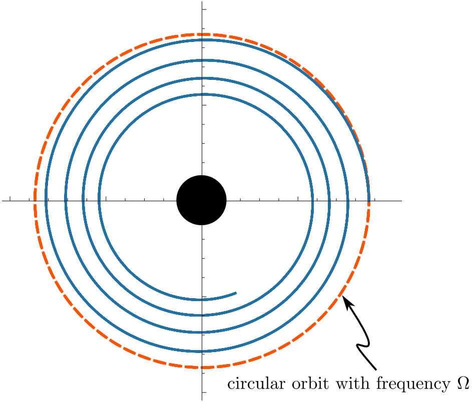
Binary parameter space



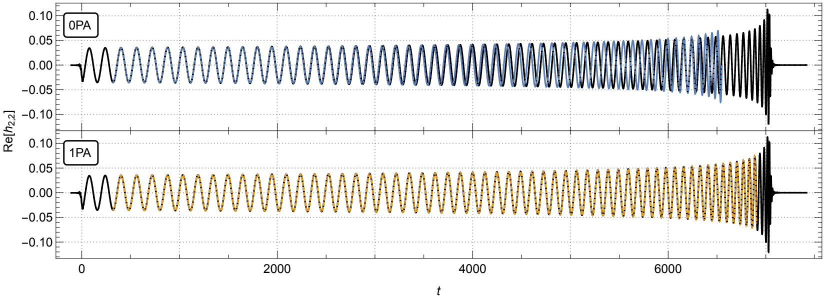
[image courtesy of Leor Barack]

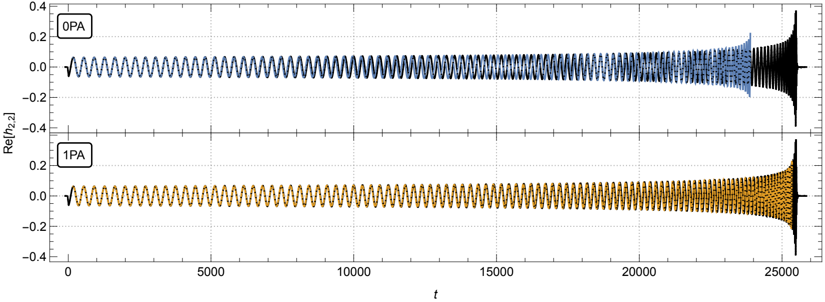
Complete 1PA for quasicircular orbits [AP, Warburton, Wardell 2013–]

- parameters: $\mathcal{J}^A = \{\Omega, M, a\}$, small a
- phase: $\frac{d\phi_p}{dt} = \Omega$
- $h^{(n)}_{\mu\nu} \sim \sum_{\ell m} h^{(n)\ell m}_{\mu\nu}(\mathcal{J}^A, r)e^{-im\phi_p}Y_{\ell m}$
- solve field equations for amplitudes $h^{(n)\ell m}_{\mu\nu}$



1PA waveforms [Wardell, AP, Warburton, Miller, Durkan, Le Tiec]





Summary

- self-force theory is currently the only viable method of modelling EMRIs
- surprisingly accurate even for comparable masses
  —use in LVK data analysis?
- can generate waveforms rapidly
- some work remains to populate parameter space for 0PA waveforms
- a lot of recent work on spinning secondaries
- only have 1PA waveforms for quasicircular, nonspinning binaries