Self-force theory and the gravitational two-body problem

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Outline

1. Lecture 1: gravitational wave astronomy, the two-body problem, and self-force theory
   - Gravitational wave astronomy: present and future
   - Gravitational self-force theory
2. Lecture 2: the local problem: how to deal with small bodies
3. Lecture 3: the global problem: orbital dynamics in Kerr
4. Lecture 4: the global problem: black hole perturbation theory

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Gravitational waves and binary systems

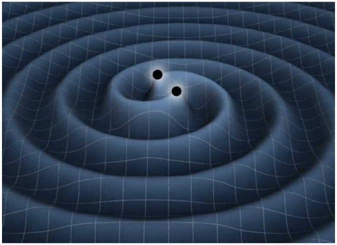


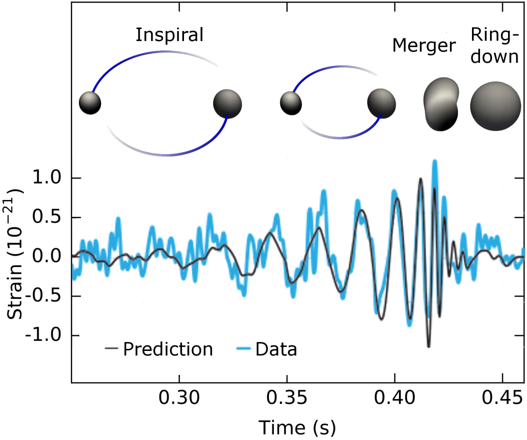
Image: Nasa/ GSFC/ The Washington Post

- compact objects (black holes or neutron stars) strongly curve the spacetime around them
- their motion in a binary generates gravitational waves, small ripples in spacetime

- waves propagate to detector
- to extract meaningful information from a signal, we require models that relate the waveform to the source

Compact binary detections

In 2015, LIGO detected gravitational waves from a black hole binary merger...



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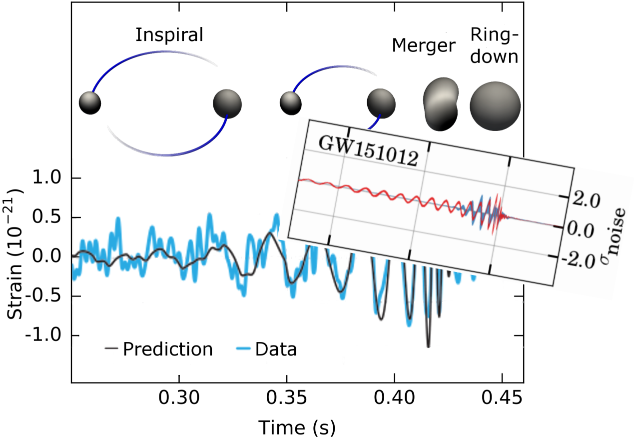

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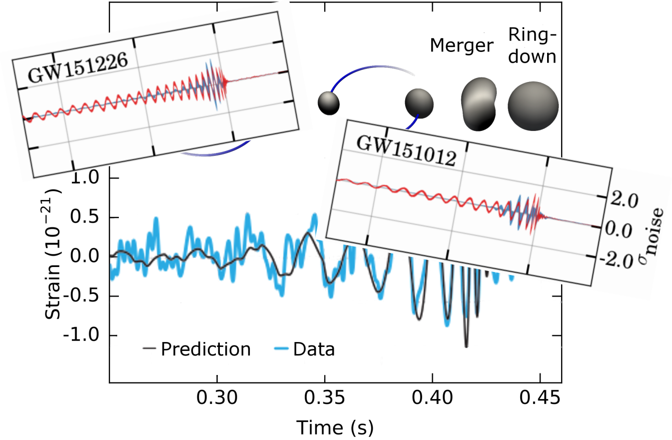

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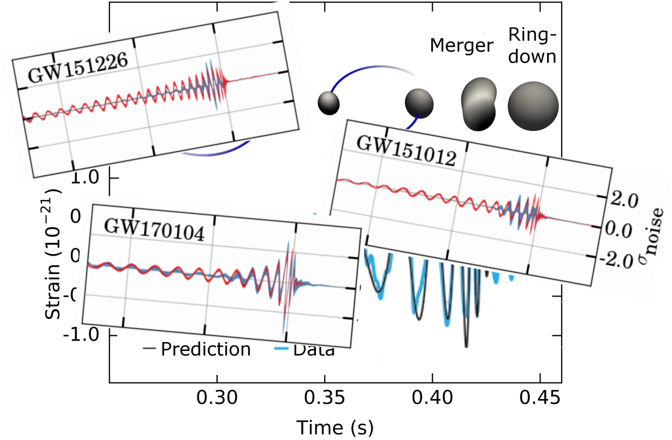

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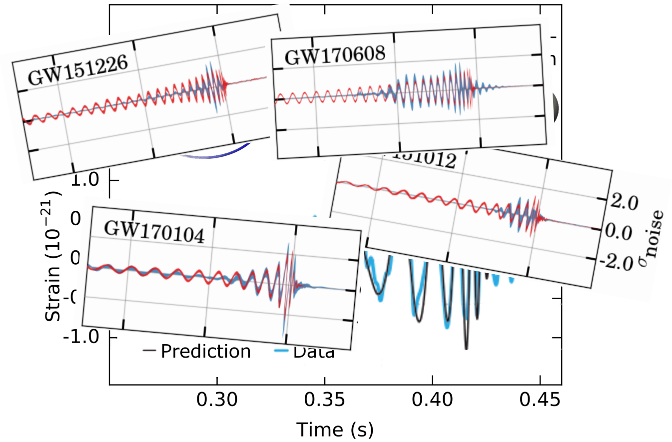

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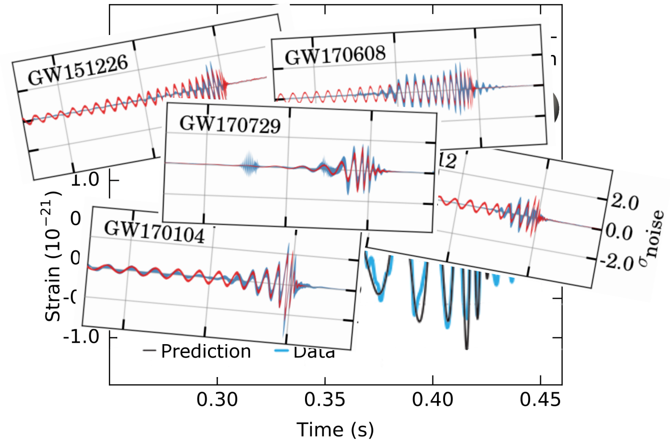

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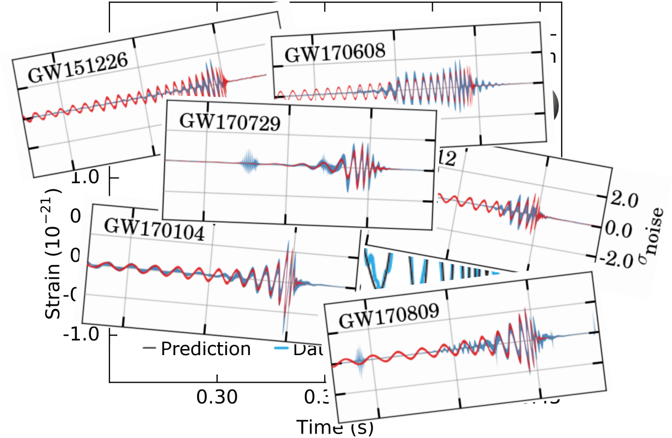

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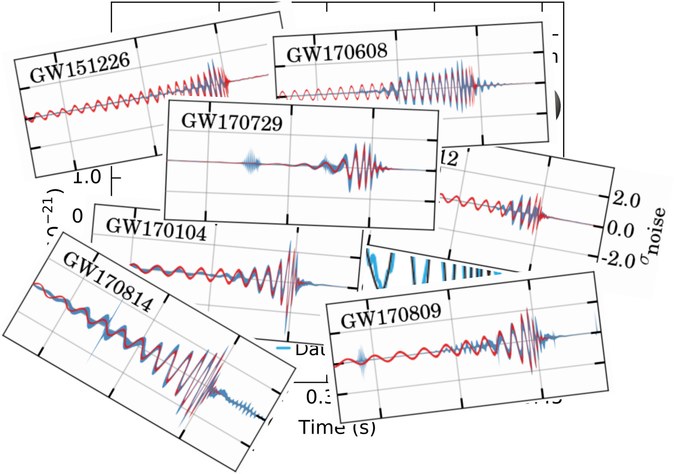

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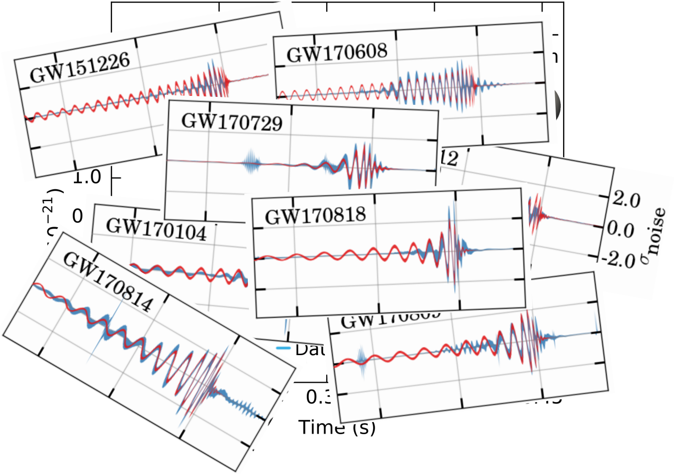

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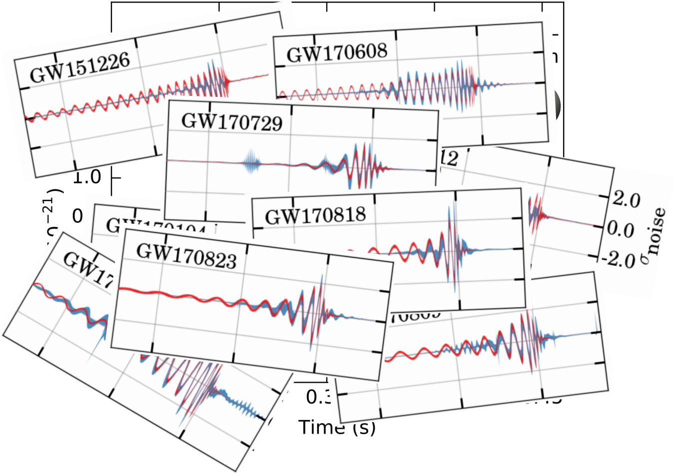
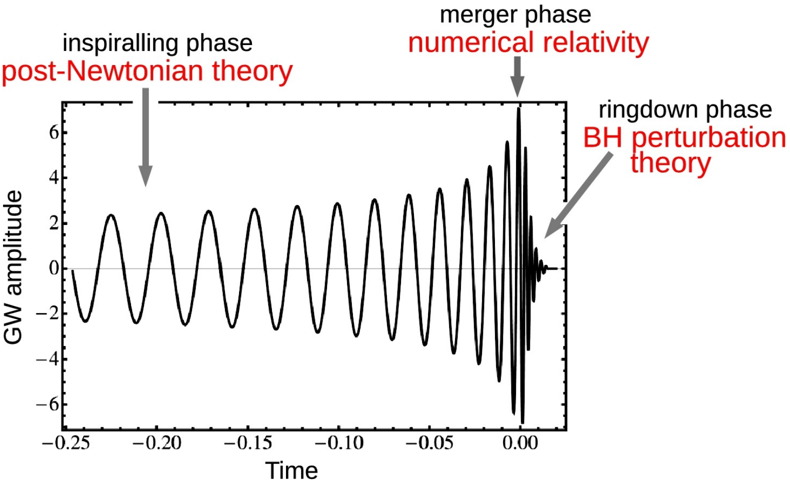


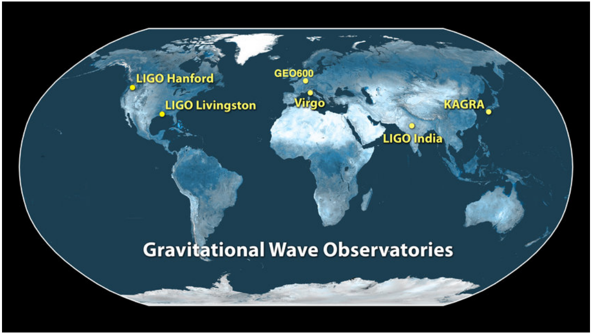
Success relies on theoretical waveforms

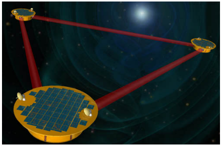
Anatomy of a typical LVK (LIGO-Virgo-KAGRA) waveform



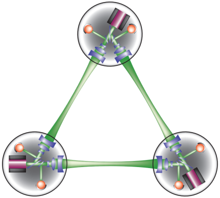
[Image credit: Luc Blanchet]

Many more detectors on the way





LISA

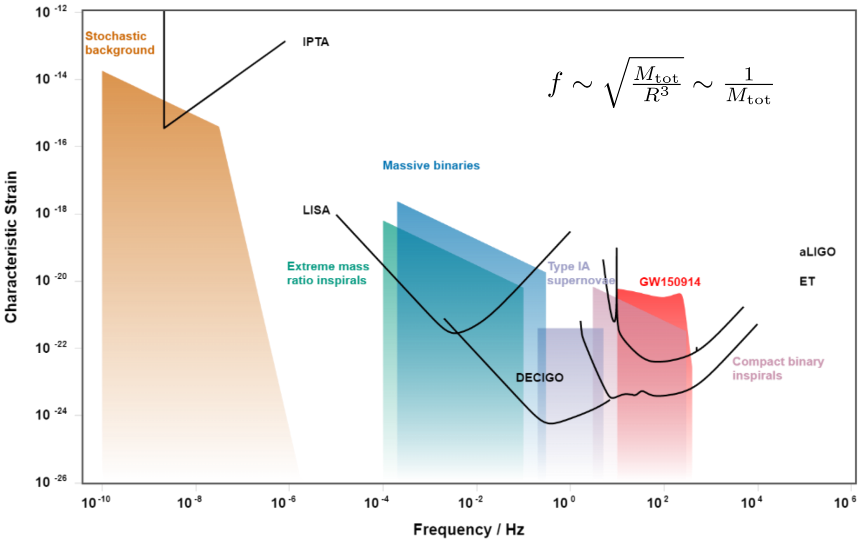


DECIGO

...and TianQin/Taiji, 3G detectors (Einstein Telescope, Cosmic Explorer),...

They'll see more types of systems, with greater precision, and further away

Many more types of signals



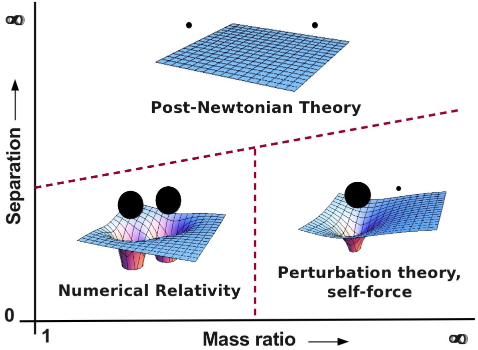
$$f \sim \sqrt{\frac{M_{\rm tot}}{R^3}} \sim \frac{1}{M_{\rm tot}}$$

http://gwplotter.com/

The gravitational two-body problem

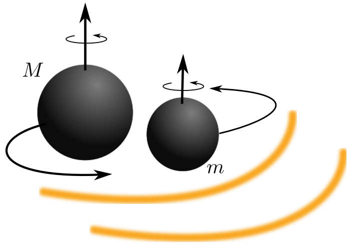
- modelling has focused on quasicircular, comparable-mass binaries
- already detecting mass ratios $\sim$ 1:25 (GW191219_163120)
- we need new and more accurate models

Binary parameter space



[Image courtesy of Leor Barack]

Comparable-mass inspirals



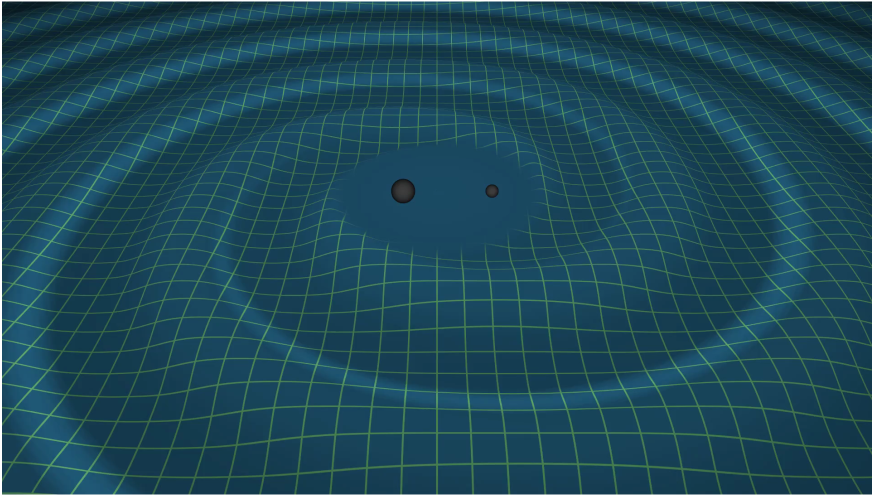
Science

- the common binaries observed by LVK
- LISA will observe earlier stages of the same binaries
- LISA will observe *massive* versions
- constrain populations and histories of BHs, NS equation of state, alternative theories of gravity

Modeling

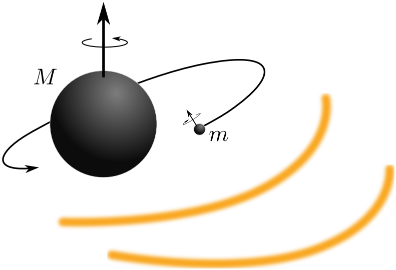
- early stages modeled by post-Newtonian (PN) theory
- late stages modeled by numerical relativity (NR)
- full evolution modeled by EOB

Typical source for ground-based detectors



[animation credit: LIGO and Virgo Collaboration]

Extreme-mass-ratio inspirals (EMRIs)

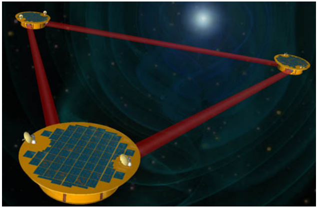


Science

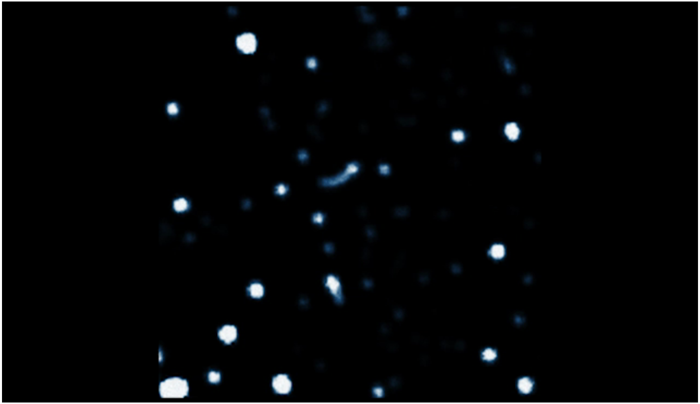
- LISA will observe extreme-mass-ratio inspirals of stellar compact objects into massive BHs
- small object spends $\sim M/m \sim 10^5$ orbits near BH $\Rightarrow$ unparalleled probe of strong-field region around BH

Modeling

- PN and NR don't work
- use black hole perturbation theory/self-force theory



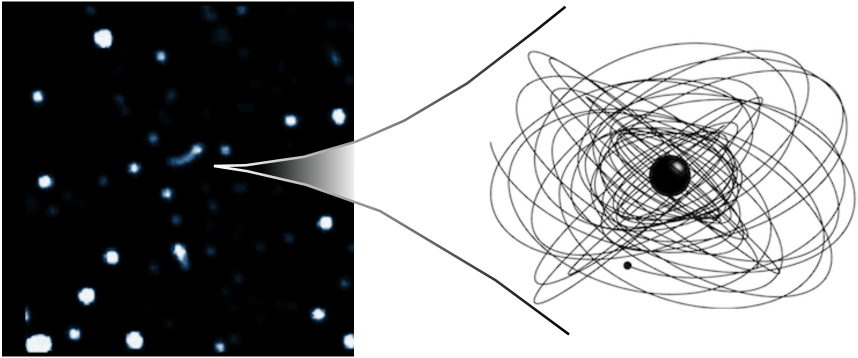
Massive BHs in galactic nuclei



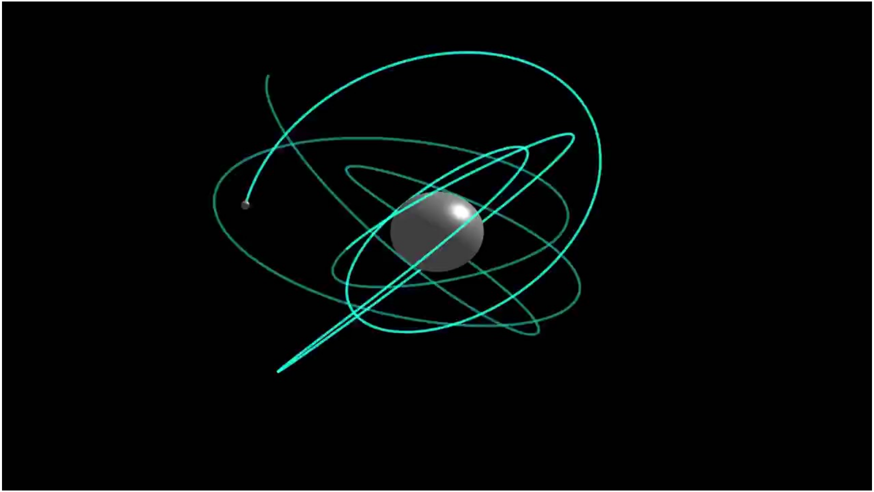
[animation credit: ESO]

closest known star $\sim 400M$ at periapsis, reaching $v \approx 0.1c$

EMRIs: probes of black hole geometry



companion spends $\sim 10^4$–10^5 orbits within LISA band, mostly within $10M$ of BH



[animation credit: Steve Drasco]

More on EMRI science

Fundamental physics

- measure central BH parameters: mass and spin to $\sim .01\%$ error, quadrupole moment to $\sim .1\%$
  $\Rightarrow$ measure deviations from the Kerr relationship $M_l + iS_l = M(ia)^l$
  $\Rightarrow$ test no-hair theorem
- measure new fundamental fields and charges, deviations from Kerr QNMs, presence or absence of event horizon, additional wave polarizations, changes to power spectrum
- constraints will generally be one or more orders of magnitude better than any other planned experiment

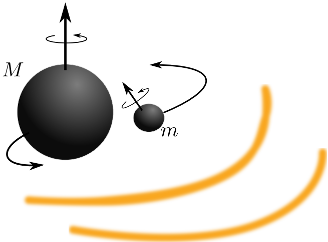
Astrophysics

- constrain mass function $n(M)$ (number of black holes with given mass)
- provide information about stellar environment around massive BHs

Cosmology

- measure Hubble constant and dark energy EOS

Intermediate-mass-ratio inspirals (IMRIs)



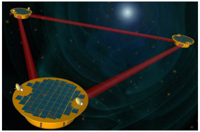
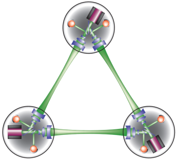
Science

- intermediate-mass BH merging with either a stellar BH or a massive BH; mass ratios $\sim 10^2$–10^4
- observable by ground-based and space-based detectors

Modeling

- current NR mostly limited to mass ratios $\lesssim 1:10$
- pushes the limits of self-force? (No!)



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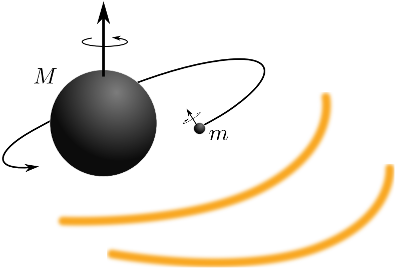
More on EMRI modeling: why self-force?



- highly relativistic, strong fields: $R \lesssim 10M$
- disparate lengthscale: m and M

- long timescale: inspiral occurs at a rate $\sim \dot{E}/E \sim m/M^2$
  $\Rightarrow$ evolution on timescale M^2/m
  $\Rightarrow$ produces $\sim \frac{M}{m} \sim 10^5$ wave cycles

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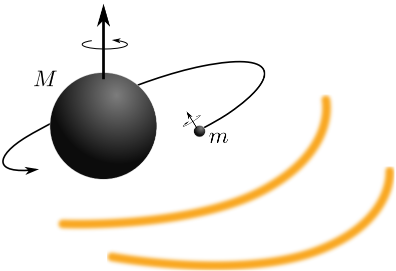

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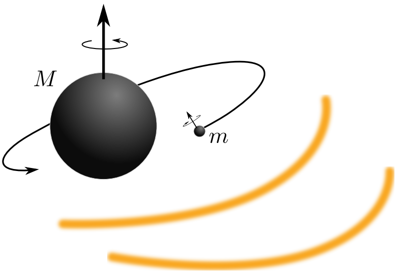

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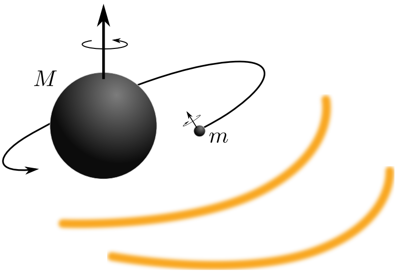

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  $\Rightarrow$ *need a model that is accurate to* $\ll 1$ *radian over those* $\sim 10^5$ *cycles*

Gravitational self-force theory

- equivalence principle: a sufficiently small and light object moves on a geodesic of the surrounding spacetime
- **but that's an approximation.** A small body perturbs the spacetime:

$$\mathrm{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu} + \dots$$

  where $\epsilon = m/M$

- this deformation of the geometry affects m's motion
  $\Rightarrow$ exerts a *self-force*

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon F^\mu_{(1)} + \epsilon^2 F^\mu_{(2)} + \dots$$

- finite-size effects also contribute to the RHS
- reduces to (and proves!) geodesic motion at zeroth order

How high order?

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon F_1^\mu + \epsilon^2 F_2^\mu + \dots$$

- force is small; inspiral occurs very slowly, on time scale $\tau \sim 1/\epsilon$
- suppose we neglect F_2^μ. Leads to error $\delta\left(\frac{D^2 z^\mu}{d\tau^2}\right) \sim \epsilon^2$
  $\Rightarrow$ error in position $\delta z^\mu \sim \epsilon^2 \tau^2$
  $\Rightarrow$ after time $\tau \sim 1/\epsilon$, error $\delta z^\mu \sim 1$

$\therefore$ accurately describing orbital evolution requires second order

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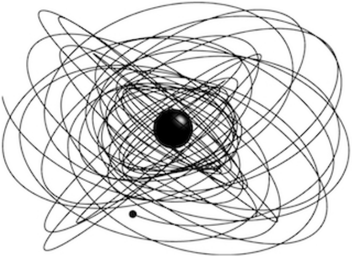
Zeroth order: test mass on a geodesic in Kerr



[image courtesy of Steve Drasco]

- geodesic characterized by three constants $J_A = (E, L_z, Q)$:
  1. energy E
  2. angular momentum L_z
  3. Carter constant Q, related to orbital inclination

- if spin isn't aligned with orbital angular momentum, then orbital plane precesses
  $\Rightarrow$ orbits are tri-periodic, with distinct radial, polar, and azimuthal periods
- phases $\varphi_A = (\varphi_r, \varphi_\theta, \varphi_\phi)$ with constant frequencies $\dfrac{d\varphi_A}{dt} = \Omega_A(J_B)$

Hierarchy of self-force models [Hinderer & Flanagan]

- self-force causes $\{E, L_z, Q\}$ to slowly evolve
  $\Rightarrow$ *two time scales*: radiation-reaction time $\sim 1/\epsilon$ and orbital time $\sim \epsilon^0$
- on radiation-reaction time, the orbital phases have an expansion

$$\varphi_A = \epsilon^{-1}\varphi_A^{(0)}(\epsilon t) + \epsilon^0\varphi_A^{(1)}(\epsilon t) + O(\epsilon)$$

- a model that gets $\varphi_A^{(0)}$ and $\varphi_A^{(1)}$ right should be enough for precise parameter extraction

Hierarchy of self-force models [Hinderer & Flanagan]

Adiabatic order

determined by

- averaged dissipative piece of F_1^μ

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Hierarchy of self-force models [Hinderer & Flanagan]

Adiabatic order

determined by

- averaged dissipative piece of F_1^μ

First post-adiabatic order

determined by

- averaged dissipative piece of F_2^μ
- rest of F_1^μ

- self-force causes $\{E, L_z, Q\}$ to slowly evolve
  $\Rightarrow$ *two time scales*: radiation-reaction time $\sim 1/\epsilon$ and orbital time $\sim \epsilon^0$
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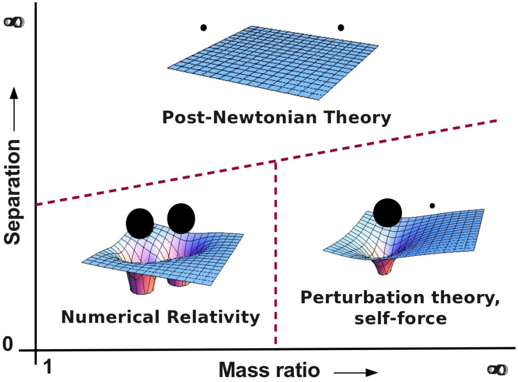
Domain of validity [van de Meent and Pfeiffer]



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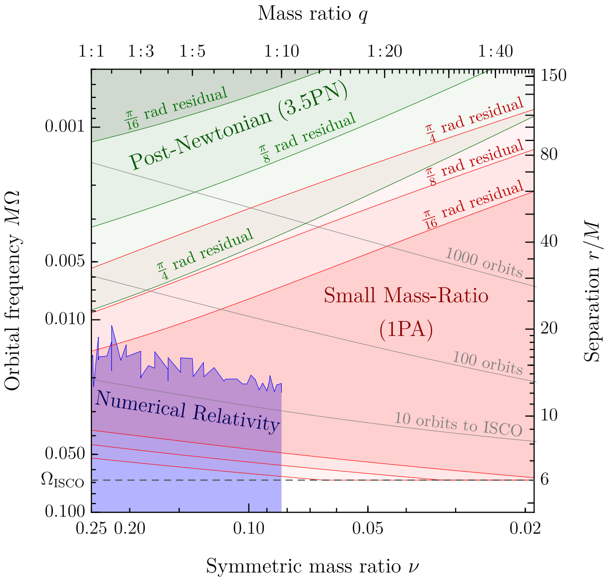
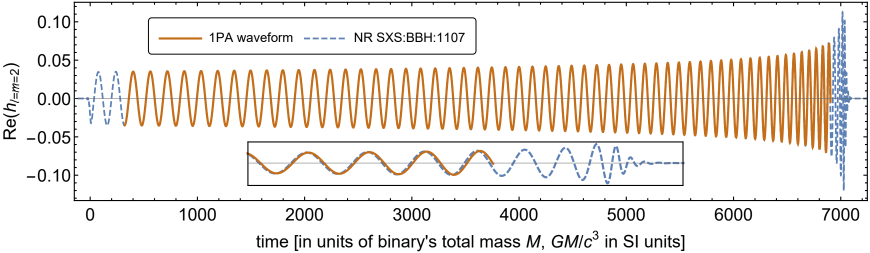


Quasicircular 1PA waveforms

Comparison with numerical relativity for quasicircular, nonspinning binary with mass ratio $\epsilon = 1/10$



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